Example Exam

Philosophical Logic

Disclaimer: This example exam aims to be representative of the final exam—though, exam difficulty is always somewhat subjective (some topics might feel more familiar or easier to you than others, etc.). Note, however, that this practice exam is *only based on material that we have covered in class so far* (up to and including chapter 4, as far as covered in class). The final exam can contain material up to and including the second-last lecture. Moreover, the practice exam might also have a different structure than the final exam (more or less exercises, different distribution of points, different topics, etc.).

The exam will be open-book, so you can access the lecture notes. But this should only *remove the need for memorizing, not the need for knowing the material*. There will not be enough time to read up on every exercise before solving it, but the notes will help, e.g., to quickly look up a formal definition.

When doing this practice exam, it is recommended to take *two undisturbed hours* for yourself, to really mimic the exam situation (which will last 120 minutes). This is because it is very different to solving exercises at home during a whole week. After having done your own attempt, you can of course discuss this example exam with others.

Exercise 1 (25 points)

1. Show that $\neg(\varphi \wedge \psi)$ is equivalent in strong Kleene to $\neg\varphi \vee \neg\psi$.
2. Show that $\neg(p \wedge q) \not\models_{\mathsf{IL}} \neg p \vee \neg q$ by using the Kripke semantics.
3. Show that $p \to q, q \to r \not\models_{\mathsf{LP}} p \to r$.

Exercise 2 (25 points) Let L be a many-valued logic in the propositional language $\mathcal{L}_{\text{prop}}$ (cf. template 3.1 of the lecture notes). Assume that among the truth-values of L are 1 and 0 such that 1 is designated, 0 is not designated, and the truth-functions corresponding to the connectives act like the classical truth-functions on classical inputs. (For example, $\mathsf{K}_3^s, Ł_3, \mathsf{LP}$ are such many-valued logics.) Show that L is a sub-logic of classical logic (i.e., $\Gamma \models_{\mathsf{L}} \varphi$ implies $\Gamma \models_{\mathsf{CL}} \varphi$).[1]

[1] This is taken from Priest (2008, sec. 7.14, exc. (2)).

Exercise 3 (25 points) Recall that, for the logic FDE, the truth table for negation is the following:

$\neg$	
1	0
b	b
n	n
0	1

Now consider an *alternative* truthtable for negation that swaps n and b:

$\neg$	
1	0
b	n
n	b
0	1

Call FDE* the many-valued logic that is defined just like FDE but uses this alternative truthtable for negation.

1. Find a sentence φ in the Boolean langauge $\mathcal{L}_{\text{bool}}$ such that $\vDash_{\mathsf{FDE}^*} \varphi$ but $\nvDash_{\mathsf{FDE}} \varphi$ (and prove this).

2. *Bonus question* (extra 5 points): Can you characterize FDE* as being a well-known logic?

Exercise 4 (25 points) Consider the fuzzy logic $Ł_\aleph$ (the version of $Ł_c$ which takes $\epsilon = 1$). So $\Gamma \vDash_{Ł_\aleph} \varphi$ iff for any fuzzy valuation $\nu : \mathsf{P} \to [0,1]$, if $\nu(\psi) = 1$ for every $\psi \in \Gamma$, then $\nu(\varphi) = 1$. Define $\varphi * \psi := \neg\varphi \to \psi$.

1. Show that, for any fuzzy valuation $\nu : \mathsf{P} \to [0,1]$, we have

$$\nu(\varphi * \psi) = \min\big(1, \nu(\varphi) + \nu(\psi)\big).$$

2. Define $p^1 := p$ and $p^{n+1} := p^n * p$. And

$$\Gamma := \{\neg p \to q\} \cup \{p^n \to q : n \geqslant 1\}.$$

Show $\Gamma \vDash_{Ł_\aleph} q$. Hint: If $\nu(p) = 0$, the first conditional gives q, and if $\nu(p) > 0$, find a large enough n such that $\nu(p^n) = 1$.

3. Show that, for any finite $\Gamma_0 \subseteq \Gamma$, we have $\Gamma_0 \nvDash_{Ł_\aleph} q$. Hint: So there is $n \geqslant 1$ with $\Gamma_0 \subseteq \{\neg p \to q, p^1 \to q, \ldots, p^n \to q\}$. Consider a valuation with $\nu(p) < \frac{1}{n}$.

This shows that $Ł_\aleph$ cannot have a sound and complete proof system (since, $\Gamma \vDash_{Ł_\aleph} q$, there would need to be a proof of q from Γ, which, qua finite object, uses only finitely premises from Γ, so we also would have $\Gamma_0 \vDash_{Ł_\aleph} q$ for some finite $\Gamma_0 \subseteq \Gamma$).[2]

[2] This is taken from Priest (2008, sec. 11.10, exc. (9)).